General Remarks.

If the theories given above are correct, a very important problem emerges from the discussion. Why does the potential across the gap vary exponentially with the time ? Already the writer has several views upon the subject, but since they lack both precision and verification they will be withheld from present publication. There can be no doubt that a close connexion exists between the value of k and the circuital constants : also, if the gas pressure or gap length is varied, a variation in k will undoubtedly result. It is suggested that there is here a fruitful field for investigation which will undoubtedly lead to a better understanding of discharge phenomena.

Acknowledgments.

I again wish to thank Professor Taylor Jones for the great interest he has taken in the work, and for his valuable encouragement. Professor Milner called my attention to the problem, and I am very grateful to him for his kindness. I am also much indebted to my wife, without whose valuable assistance the rather laborious calculations would have taken double the time to perform. The experimental work was performed in the Research Laboratories of the Natural Philosophy Department of the University of Glasgow.

References.

(1) Phil. Mag. xiii. p. 824 (1932).
(2) Phil. Mag. November 1912.
(3) *Ann. d. Phys.* xxxvi. p. 281 (1911).

December 1932.

LVI. *Some Elementary Considerations connected with Modern Physics.—*I. *The Larmor-Lorentz Transformation.* *By* Sir Oliver J. Lodge, *F.R.S.*, *and* Professor Alfred Lodge, *M.A.**

LITERARY people, besides pursuing their discoveries and producing their masterpieces, sometimes play about with their subject as if they enjoyed it; and thereby they consolidate and make palatable and intelligible what

* Communicated by Sir Oliver Lodge.

might otherwise be rather arid and esoteric. Literary and artistic criticism have become an occupation in themselves, and commentaries serve a useful purpose. There is very little play in modern physics. The subject is difficult, the ground is new, and workers are keen, making up for their fewness by concentration and industry. Repetition and gleaning seem comparatively frivolous while the main crop is waiting to be garnered in, and new adventures are calling in every direction. Yet a time for consolidation and digestion must come, and novelties require to be assimilated as well as discovered.

With this preamble we will deal with a few elementary matters.

Everyone must have noticed that the composition of relative velocities has the same form as the composition of a trigonometrical function of two angles, especially of imaginary angles; and sometimes trigonometrical expressions are neater than algebraic ones. Now Relativity insists first of all that there is in nature a constant absolute velocity c, a characteristic of space or of the medium which fills space, which turns out to be of the same order, and perhaps of the same value, as the undoubted velocity with which space transmits periodic transverse disturbances. Furthermore, whatever may happen to waves under exceptional circumstances such as those in electrified space, the velocity of a particle or of a quantum of energy can never exceed c, and indeed can only approach it asymptotically; so that a particle's velocity bears to c a ratio which may approach, but cannot exceed, unity. (While at the same time that factor of the particle's motion which confers on it momentum tends to increase without limit.)

A trigonometrical function with this property is well known, namely, the hyperbolic tangent of an angle; or, what is the same thing, the imaginary circular tangent of an imaginary angle. So let us represent $\frac{u}{c}$ as $\tanh\theta$, and see what happens to the momentum factor m_0, and to the familiar Larmor-Lorentz transformation.

The FitzGerald contraction $\frac{1}{\beta}$ or $\sqrt{1-\frac{u^2}{c^2}}$ becomes $\sqrt{1-\tanh^2\theta}$, that is $\operatorname{sech}\theta$. Hence β is simply $\cosh\theta$, a function which is readily pictured as the curve of a

hanging chain. So that β rises from unity when $\frac{u}{c}$ is small, as it usually is, and tends towards infinity as $\frac{u}{c}$ approaches unity.

Accordingly, taking an auxiliary angle for subsequent interpretation, and putting $\beta=\cosh\theta$, and $u=c\tanh\theta$, the conditions are satisfied; and, moreover, the transformation of the coordinates of a particle moving with velocity u, or of a particle referred to moving axes, is :—

$$\left.\begin{aligned} x'&=\beta(x-ut)=x\cosh\theta-ct\sinh\theta,\\ \text{and}\qquad ct'&=\beta(ct-ux/c)=ct\cosh\theta-x\sinh\theta,\end{aligned}\right\} \quad . \quad (1)$$

which equations are identical with the old established form for transition from one set of axes to another set rotated through the angle $i\theta$, namely :—

$$\left.\begin{aligned} x'&=x\cos i\theta+y\sin i\theta,\\ y'&=y\cos i\theta-x\sin i\theta,\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (2)$$

where

$$y=ict, \quad \text{and} \quad y'=ict'.$$

This identity at once suggests that the Larmor-Lorentz transformation to moving axes is equivalent to a rotation of the axes about an imaginary angle $i\theta$. We do not properly know what relative locomotion such as u really is; absolute motion through the ether may very well involve something equivalent to rotation.

The problem is whether any physical significance may be attached to this rotation. It is known that time can be converted into space by multiplying it by the factor ic. And if there is a constitutional rotation in the ether it may very well be represented by an imaginary angle $i\theta$, since it is clearly not an ordinary revolution in space, and its real physical interpretation is not expressible in space relations without the converting factor i. Time cannot be real space: ict is only space of a fourth dimension, the factor i being appropriate for expressing a dimension at right angles to the three simple space dimensions. It represents rotation through a right-angle, and thus enables us to pass from three to four dimensions and to include time.

But ordinary rotation is an affection of matter. What rotation can mean in ether we do not know; it presumably

has an analogy with material rotation, and might reasonably be prefixed by a $\sqrt{-1}$. This mode of separating ether terms from matter terms may be serviceable, for it will keep them from being mixed up. A decay of matter e^{-nt} may impart to the ether an oscillation e^{-int}. The rate of decay would thus determine the frequency of the consequent radiation. That is to say, a disappearance of matter at the rate h might give rise to radiation hv.

To get the new or transformed velocity appropriate to a constant u when the old velocity $\frac{dx}{dt}$ is v, that is to compound v and u, we must reckon the value of $\frac{dx'}{dt'}$; and that comes out thus :—We can take v as $c \tanh \phi$, another auxiliary angle, and since by (1)

$$dx' = \cosh \theta\, dx - c \sinh \theta\, dt$$

and

$$c\,dt' = c \cosh \theta\, dt - \sinh \theta\, dx,$$

it follows that

$$\frac{1}{c} \cdot \frac{dx'}{dt'} = \frac{v \cosh \theta - c \sinh \theta}{c \cosh \theta - v \sinh \theta}$$

$$= \frac{\tanh \phi \,.\, \cosh \theta - \sinh \theta}{\cosh \theta - \tanh \phi \sinh \theta}$$

$$= \frac{\sinh \phi \cosh \theta - \cosh \phi \sinh \theta}{\cosh \phi \cosh \theta - \sinh \phi \sinh \theta}$$

$$= \frac{\sinh(\phi - \theta)}{\cosh(\phi - \theta)} = \tanh(\phi - \theta).$$

So to summarize:

$$\frac{dx}{dt} = c \tanh \phi$$

and

$$\frac{dx'}{dt'} = c \tanh (\phi - \theta),$$

where

$$u = c \tanh \theta.$$

The Larmor-Lorentz Transformation—Graphically Treated.

Let us begin again.

Imagine two observers, one of whom moves relatively to the other with a positive velocity u along a direction

which we shall choose as the axis of x, and let them observe an interval between two events separated both in space and time ; and let x, t be the estimates of distance (along the x-axis) and of time, made by one observer, while x', t' are the corresponding estimates made by the observer who is moving with the above velocity u. Then the equations connecting these two pairs of quantities are known to be

$$x'=\beta(x-ut), \qquad t'=\beta\left(t-\frac{ux}{c^2}\right),$$

$$x=\beta(x'+ut'), \quad t=\beta\left(t'+\frac{ux'}{c^2}\right),$$

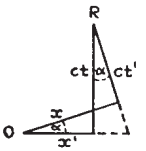
where c denotes an ultimate velocity unsurpassed by any material thing, and β is a coefficient required to make the second pair of equations consistent with the first pair, and therefore with the principle of relativity.

To find β, substitute in the second pair the values of x' and t' from the first pair. Then

$$x=\beta^2\left(x-ut+ut-\frac{u^2x}{c^2}\right)=\beta^2x\left(1-\frac{u^2}{c^2}\right)$$

and $$t=\beta^2\left(t-\frac{ux}{c^2}+\frac{ux}{c^2}-\frac{u^2t}{c^2}\right)=\beta^2t\left(1-\frac{u^2}{c^2}\right).$$

Each of these identities requires β to be equal to $\left(1-\frac{u^2}{c^2}\right)^{-\frac{1}{2}}$. Therefore $\frac{1}{\beta}$ is the FitzGerald contraction.



To represent these graphically, let $u=c\sin\alpha$, whence $\beta=\sec\alpha$.

$$\therefore \qquad \left.\begin{aligned} x'&=x\sec\alpha-ct\tan\alpha,\\ ct'&=ct\sec\alpha-x\tan\alpha,\end{aligned}\right\}$$

and

$$\left.\begin{aligned} x&=x'\sec\alpha+ct'\tan\alpha,\\ ct&=ct'\sec\alpha+x'\tan\alpha,\end{aligned}\right\}$$

equations which are represented by the accompanying figure.

These equations satisfy the equation $(ct)^2-x^2=(ct')^2-x'^2$, which is independent of α, and is therefore the same for different values of the relative speed u of the two observers, and therefore invariant for all observers. If, therefore, x and ct are represented in a diagram as the rectangular coordinates of a point P, and x' and ct' are the coordinates of a point P', this invariance indicates that P and P' lie on the same hyperbola for all values of α.

To find x', ct' when x, ct, and also the angle α, are given.

Take one event at the origin ,where we may suppose the observers to be located, one at rest, and the other having relative velocity u along the axis of x ; and let the second event be represented (to the first observer) by a point P (fig. 1) whose coordinates are (x, ct). The problem is to find x' and ct' for the given value of $\alpha=\sin^{-1}\frac{u}{c}$. With centre O, radius ct, describe a circle, and draw the radius OT at the angle α with the y axis. Draw the tangent TQ cutting PN the ordinate of P in Q. Then

$$\mathrm{QN}=ct', \quad \text{and} \quad \mathrm{TQ}=x'.$$

For, if QT cuts the y-axis at R, and QM is parallel to ON, it is clear that

$$\begin{aligned}\mathrm{TQ}&=\mathrm{RQ}-\mathrm{RT}\\&=x\sec\alpha-ct\tan\alpha\end{aligned}$$

and

$$\begin{aligned}\mathrm{NQ}&=\mathrm{OR}-\mathrm{RM}\\&=ct\sec\alpha-x\tan\alpha.\end{aligned}$$

Also it is clear that

$$\begin{aligned}x(=\mathrm{MQ})&=\mathrm{TQ}\sec\alpha+\mathrm{OM}\tan\alpha\\&=x'\sec\alpha+ct'\tan\alpha,\end{aligned}$$

and

$$\begin{aligned}ct(=\mathrm{OT})&=\mathrm{OM}\sec\alpha+\mathrm{TQ}\tan\alpha\\&=ct'\sec\alpha+x'\tan\alpha.\end{aligned}$$

And

$$\mathrm{OQ}^2=(ct)^2+x'^2=(ct')^2+x^2,$$

so that

$$(ct)^2-x^2=(ct')^2-x'^2.$$

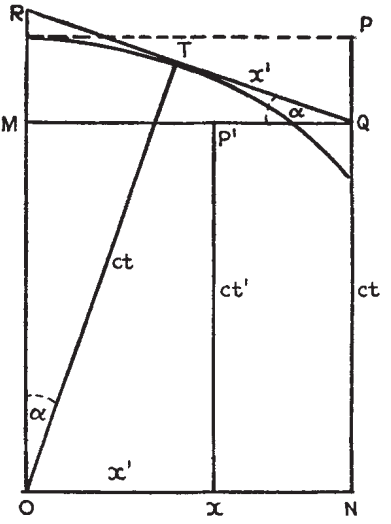
If now MP' is taken equal to TQ, on the line MQ, the coordinates of P' are (x', ct').

So P' represents the occurrence of the event P as it

appears to an observer at O who is moving along the x axis with a speed $u=c \cdot \sin \alpha$.

The value of the invariant quantity $(ct)^2-x^2$ is positive, zero, or negative according as $ct>$, $=$, or $<x$. Its geometrical significance in each case can be illustrated by taking three points A, B, C having the same value of ct, but with values of x respectively less than, equal to, and greater than ct. We will find the corresponding values A′, B′, C′ for a given value of α, and then consider their loci as α changes. The diagram (fig. 2) also shows

Fig. 1.



the parallelogram into which the rectangle OYCc is distorted.

The sides of this parallelogram are inclined to the corresponding sides of the rectangle at an angle γ such that $\tan \gamma = \sin \alpha$. For consider the lines AC, A′C′, and let the coordinates of A and C be (x_1, ct) and (x_2, ct), the value of t being the same for both A and C, but the values of x being different ; then, since

$$x' = x \sec \alpha - ct \tan \alpha ;$$
$$ct' = ct \sec \alpha - x \tan \alpha ;$$

$$\left.\begin{aligned} \therefore \quad x_1' - x_2' &= (x_1 - x_2) \sec \alpha \\ \therefore \quad ct_1' - ct_2' &= (x_2 - x_1) \tan \alpha \end{aligned}\right\} \text{as } ct \text{ is constant};$$

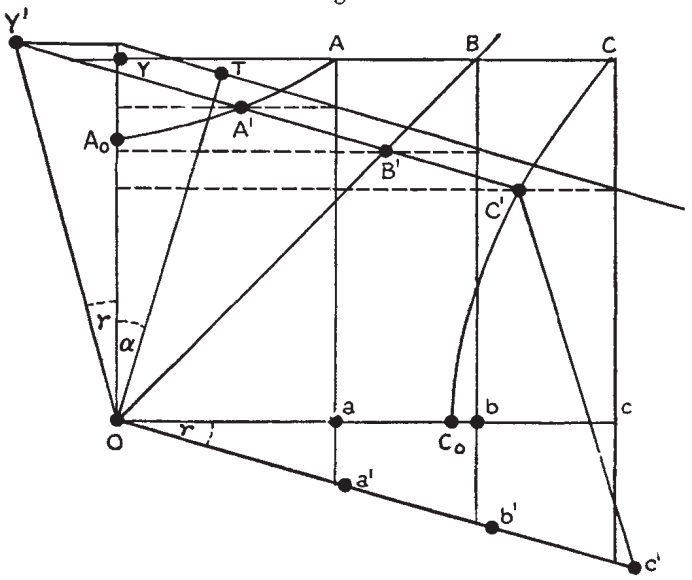
hence, if γ is the angle between AC and A′C′, which is the same as the angle between Oa and Oa',

$$\tan\gamma = \frac{\tan\alpha}{\sec\alpha} = \sin\alpha = u/c.$$

Similarly, the angle YOY′ is also equal to γ.

So, in general, the derived or projected figure is distorted angularly as well as in length, but the ratio of A′B′ : B′C′ is equal to the ratio of AB : BC, and parallel lines remain parallel.

Fig. 2.



The points AA′ lie on a hyperbola whose semi-transverse axis is OA_0, A_0 being the point where the circle, centre a, radius aA, cuts the y axis. This is the locus of A′ as α varies. Similarly, the locus of C′ is the hyperbola having OC_0 as its semi-transverse axis, C_0 being the point where the circle, centre Y, radius YC, cuts the x axis.

The 45° line, containing B and B′, is an asymptote of the system of hyperbolæ, and the other asymptote is perpendicular to it.

There are two directions in which there is no angular distortion, viz., if lines are drawn parallel to the asymptotes their representatives in the system, or, as we may

call them, their projections, are also parallel to the asymptotes, but they will be altered in length.

For

$$x' = x \sec \alpha - ct \tan \alpha,$$
$$ct' = ct \sec \alpha - x \tan \alpha$$
$$\therefore \quad x' + ct' = (x + ct)(\sec \alpha - \tan \alpha)$$
and
$$x' - ct' = (x - ct)(\sec \alpha + \tan \alpha).$$

Therefore, if $x+ct$ is constant, $x'+ct'$ will also be constant, but a smaller constant ; and if $x-ct$ is constant, $x'-ct'$ will be a bigger constant.

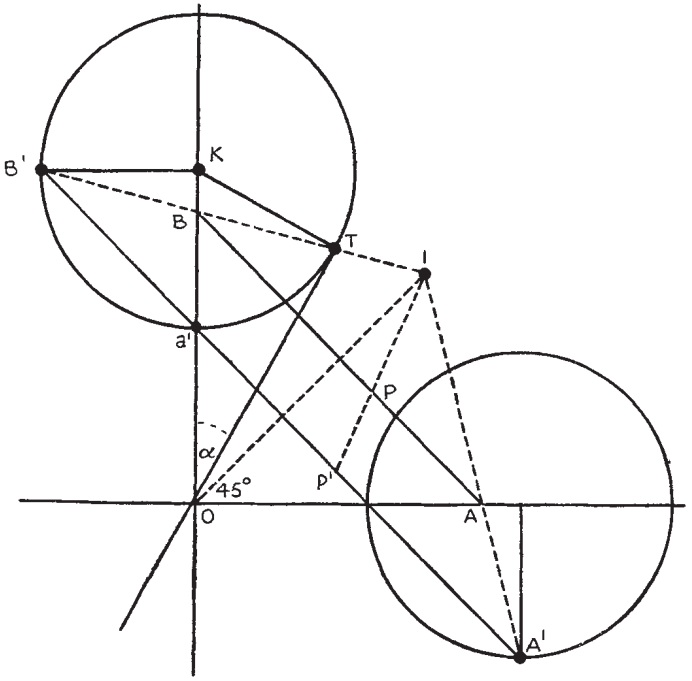
The importance of these lines lies in the fact that they are the time-distance tracks of light travelling from an event towards the observers, assuming that the light is travelling with the velocity c. The lines $x+ct=$constant emanate from points on the right of the diagram, when x is positive, and the lines $x-ct=$constant come from points on the left. We shall consider only points on the right, utilizing the lines $x+ct=$constant. It is important to see that the light-tracks from (x', ct') are parallel to those from (x, ct) in accordance with the condition that light travels with the same velocity c whatever may be the relative speed u.

The accompanying diagram (fig. 3) shows two corresponding tracks, AB and A′B′, the one starting from A or from any event P on the line AB, and the other starting from A′ or from a point P′ corresponding to P. The dotted lines in the figure show one method of finding P′ when P is given. This construction is based on the fact that P′ must divide A′B′ in the same ratio as P divides AB. The distance OB shows the time at which the observer will see the event at A if $u=0$, and Oa' shows the time at which he will see the same event if the relative velocity is $u=c \sin \alpha$. In each case OB is the observer's time track : the diagram is so adjusted that the velocity u is given to the object on which the event occurs. This will be alluded to again and exemplified in the sequel.

Although an attempt is made in these diagrams to depict time as if it were space, remember that a time-diagram cannot be static. It inexorably progresses, turning the future into the past. Consequently the line through the observer has to move upwards : or else the whole diagram has to move downwards past O. Events

are not seen when they actually occur; an observer has to wait till the light from them reaches him. The velocity of light in these diagrams is represented by the slope of 45°, since the distance travelled by the light in time t is equal to ct, so that $\frac{dx}{c\,dt} = \pm 1$ according to its direction of travel. The light-line, therefore, from an event P is a line of 45° towards the y axis, *i. e.*, towards

Fig. 3.



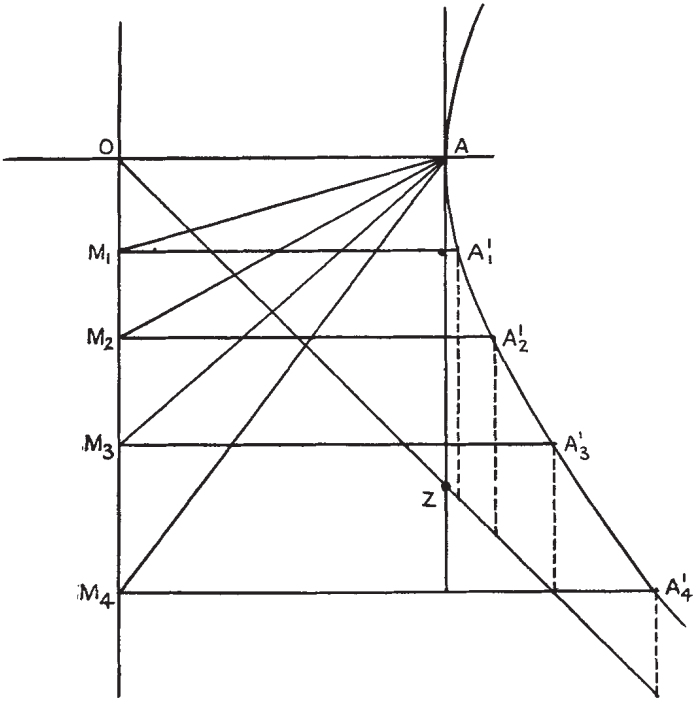
the line $x=0$ along which the observer is travelling in time.

Consequently an event occurring at P (x, ct) is seen at time T given by $cT=x+ct$ by the stationary observer, and is seen by the other observer at time T′ given by the equation $cT'=x'+ct'$. Note that the moving observer, considering himself at rest, has placed P′ in his diagram so that the axes of x' and t' coincide with those of x and t.

Scale of Diagrams.—We have assumed the scales of x and ct to be equal, so that when $x=ct$ the line is at 45°.

As these diagrams are most useful when applied on a cosmic scale, it will be better really to measure x and ct both on the light-time scale, *i. e.*, really to measure x by the time light would take to travel the distance, so that we really depict $(x/c, t)$ rather than (x, ct). It is merely, then, a matter of scale : in either case the light-tracks are at 45° in the map.

Fig. 4.

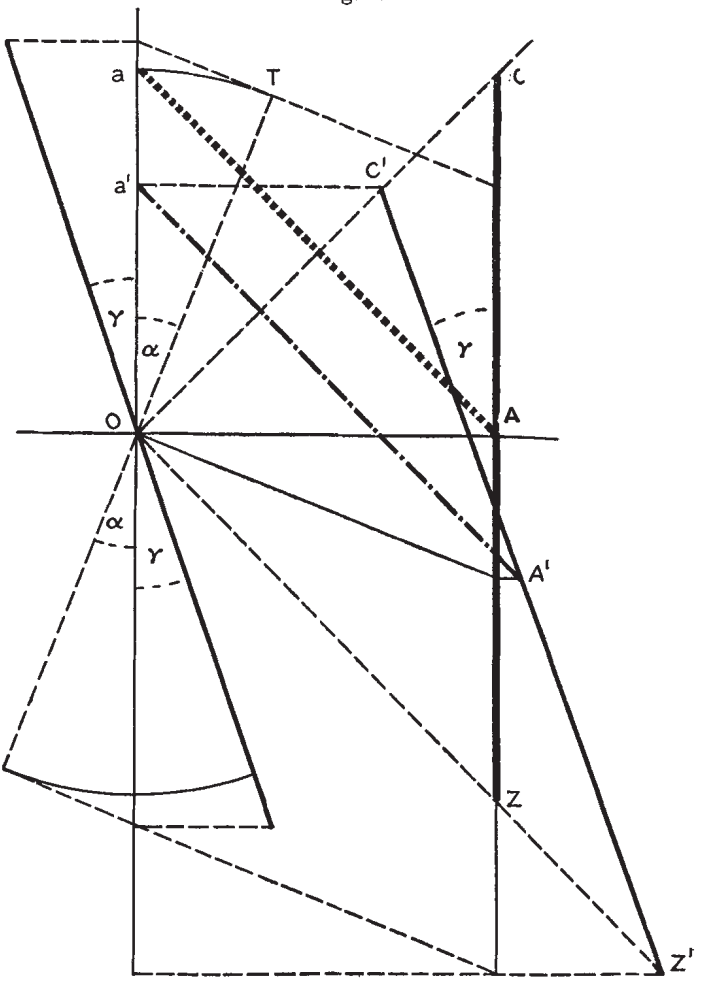


Application to Astronomy.

The geometry indicated in the preceding pages can be utilized for the purpose of what may be called a celestial time-distance map of, for instance, the planets as seen from the earth. Take, for instance, the planet Jupiter and an event on it such as an eclipse of one of its satellites. Instead of considering two observers, one at a fixed distance from Jupiter and the other moving towards it with velocity u, take one observer, and

consider the time-distance track of Jupiter according as it is at a stationary distance or is approaching the observer with velocity u, and on this track show where and when the eclipse would appear to take place in the

Fig. 5.



two cases. There are two distinct times in each case: (1) when the eclipse *occurs*, (2) when it is *seen*. The eclipse occurs at time t' or t according as there is or is not relative motion; the eclipse is seen when $T' \equiv t' + \frac{x'}{c}$ or $T \equiv t + \frac{x}{c}$ under the same variety of conditions.

In fig. 5 the time-tracks of Jupiter are ZAC for the case when its distance from the observer is constant, *i. e.*, when $u=0$, and Z′A′C′ for the case when it is moving towards the observer with velocity u. The angle that this latter track makes with AC is γ, where $\tan\gamma=\frac{u}{c}$. This angle is the same as would be given for such a track in any Euclidean distance-time chart, its gradient being proportional to its velocity : the only peculiarity is its displacement in space, as it is not drawn from A as in an ordinary track such as in railway time-tables, but from A′, a point earlier in time than A, and further away from the observer, but so placed that the track A′C′ crosses the line OA in front of A.

If the eclipse occurs at A when the distance x remains constant, the diagram puts it at A′ if Jupiter is approaching O with velocity u. This enables the observer in this latter case to see the eclipse earlier, viz., after an interval Oa' instead of Oa, although the light travels at the same rate in either case : the light-tracks Aa, A′a' are parallel, but not coincident.

Note.—It is not necessary actually to draw these light-tracks to find the time-intervals : they can equally be reckoned from the distances of A, A′ from the 45° line OZZ′, measured in the time direction, since these are parallel and equal to Oa, Oa' respectively. Fig. 4 shows this for different observers, or for one observer with different speeds.

We have shown that the estimated time-intervals between the *occurrence* of an event and its being *seen* are indicated by its time-distance from the asymptote through the observer ; and that these time-distances, shown in the diagram by AZ and the parallel dotted lines from the position of A′ (a position which is always on the hyperbola whose vertex is at A), are less and less according as the relative velocity u becomes greater and greater.

So, if there is a second event occuring later (not shown) its estimated time-interval will also be proportional to these distances. It does not matter whether the distance of Jupiter remains the same : it will not if it is really approaching : the figure will be *similar* in all respects to the one drawn.

So, the time-interval between two events as estimated by a moving observer will depend on the value of u, and

will be proportional to the distances of the various A′ points from the asymptote. As u increases, these intervals become less and less, as is rather distinctly shown by this hyperbola diagram (fig. 4).

[The mode of finding these points is shown by the construction already stated, viz., making MA′=MA= OA sec α, where α is OAM.]

We are taking the eclipse as occurring at time $t=0$, *i. e.*, "now" to one observer; it is the most interesting case, but in all cases the intervals between occurring and being seen obey similar laws to those indicated above, for all events are "now" at some time or another!

Recapitulation, with special reference to Fig. 5.

Let (x, ct) denote the distance away, and the time of occurrence, of an event, when the distance of the observer from the place where the event occurs is constant. It does not follow that the observer sees it then, since light takes time to travel the distance. In the diagram he has to wait till the light-line arrives at $x=0$. So all events are seen in the future if they lie above the $-45°$ line OZ. Taking the line OX as the "now" line, the line of zero time, events below this line have already occurred, but are *seen* in the future. So OZ is the line of simultaneous visibility, though OX is the line of simultaneous occurrence.

Now if the event is occurring at a place which is approaching the observer with velocity u, its occurrence appears to be earlier and at a slightly greater distance, but any event which would have been seen in the future when $u=0$ will still be seen in the future, though at an earlier moment; since any event A, which lies above OZ when $u=0$, will still lie above it whatever u may be, as long as u is less than c.

So far we have considered that, when Jupiter approaches the observer, an event appears to occur earlier than if it were at relative rest. In the diagram (fig. 5) the time axis is fixed, and the path of Jupiter is tilted instead. This tilt is $\gamma=\tan^{-1} u/c$. The apparently earlier occurrence is certainly indicated by the diagram and seems to be justified by the equations. But in what follows we endeavour to show that this curious time distortion, so repugnant to common sense, need not necessarily be considered to be a physical fact, though it still remains a

useful mathematical fiction. If we can permit ourselves to imagine that light may reach the eye of an approaching spectator with a speed $c+u$, we shall see that it is possible to consider the time of occurrence as perfectly definite whatever the relative motion may be.

Seeing an Event occurring now.

To reckon the times, T and T′, of seeing an event after its occurrence, i. e., *after the time when* $t=0$.

If the object on which the event occurs is at a constant distance x from the observer, then, when $t=0$, the time required before the event is seen is

$$\mathrm{T}=\frac{x}{c}.$$

But if the observer is approaching the object with velocity u, i. e., if

$$\frac{dx'}{dt'}=-u,$$

the time at which the event will be seen will be T′, where

$$\mathrm{T}'=t'+\frac{x'}{c}.$$

When $t=0$, the equations

$$x'=x\sec\alpha-ct\tan\alpha,$$
$$ct'=ct\sec\alpha-x\tan\alpha,$$

become

$$x'=x\sec\alpha,\quad ct'=-x\tan\alpha;$$

$$\therefore\quad \mathrm{T}'=\frac{x}{c}(\sec\alpha-\tan\alpha).$$
$$=\frac{x}{c}\cdot\frac{1}{\sec\alpha+\tan\alpha}$$
$$=\frac{x}{c}\cdot\frac{\cos\alpha}{1+\sin\alpha}$$
$$=\frac{x\cos\alpha}{c+u}$$

This expression is actually less than $\frac{x}{c+u}$, which would have been its Euclidean value under a joint speed of

c and u. It *is* this fraction, with $x\cos\alpha$ instead of x, i. e., with the FitzGerald contraction applied to x. (To a first approximation the two values are equal, since $1-\cos\alpha$ is of the second order of small quantities compared with $\sin\alpha$.)

Referring to fig. 6, if the event A occurs on an object (*e. g.*, Jupiter) which is at a fixed distance from the observer at O, he will see the event at S, *i. e.*, when the light reaches S. If, however, the object is approaching the observer with velocity u, he will see the event at S′, *i. e.*, when the light reaches S′.

A′ is a point on the rectangular hyperbola of which A is the vertex, and the space-time track of the moving object, viz., A′BC′ is the tangent at A′ to the hyperbola, and slopes at an angle γ with the time axis OS, where $u=c\tan\gamma=c\sin\alpha$.

If this tangent cuts OS at T, the line TA makes an angle α with OS, and is perpendicular to MA, where

$$\mathrm{MA}=\mathrm{MA}'=x'=x\sec\alpha,$$

and also

$$\mathrm{OA}=x,\quad \mathrm{t}=0,$$

$$\mathrm{OB}=x\cos\alpha$$

$$\mathrm{OS}'=x\sec\alpha-x\tan\alpha$$

$$=c\,.\,\frac{x\cos\alpha}{c+u}=c\mathrm{T}'.$$

S″ is the seeing point if the object is moving away with the same velocity, the positive time direction being downwards in this case, using the same diagram.

$$\mathrm{OS}''=x\sec\alpha+x\tan\alpha$$

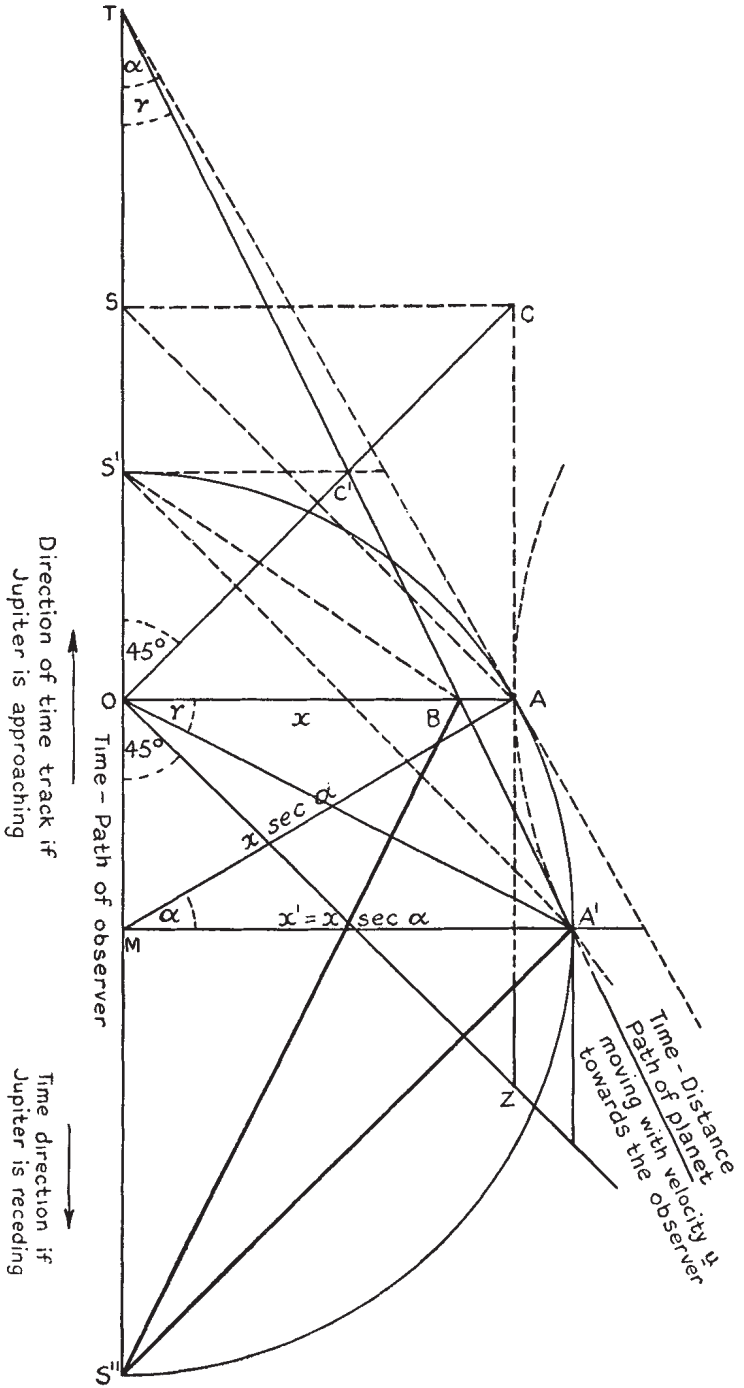
and

$$\mathrm{T}''=\frac{x\cos\alpha}{c-u}.$$

It is possible for all observers to agree when an observed event really occurred.

Now let us see what difference the relative velocity of the object and observer makes. An event which really occurs at A seems to occur either at A′ or at B; that is, it seems to occur earlier and a little further away than it actually did, or else time remains unaltered, but distance is foreshortened; that is, we interfere either

Fig. 6.


T
α
γ
S
C
S'
C'
45°
O
γ
x
B
A
45°
Time - Path of observer
x sec α
α
x' = x sec α
M
A'
Z
S''
Direction of time track if Jupiter is approaching
Time direction if Jupiter is receding
Time - Distance
Path of planet moving with velocity u towards the observer

with time and distance or with distance only. But the interference with time introduces the artificiality of making the event occur before it does; while the disturbance of distance is only extending the FitzGerald contraction till it is of cosmic importance. A′ is best in a diagram, but philosophically B is better, since time is not interfered with, and distance is only altered in a way to which we are accustomed. No matter whether the event seems to occur at A′ or at B, it is seen at the same time. A′ is an inference, just as B is. But B has the remarkable advantage of being in accord with common sense. For the time taken before the event is seen by the moving observer is $\frac{x \cos \alpha}{c+u}$, that is, the velocity of the observer is added to the velocity of light, and as the two are approaching each other, this is what one would expect by common sense. The velocity of light is, of course, constant, but measured by a moving observer comes out $c \pm u$ according as the observer is approaching towards or receding from the source of light.

This is in agreement with common sense, but is not in accord with orthodox relativistic teaching. By it a *Doppler effect* is given to the moving observer, but the *velocity* as measured is supposed to be the same as if he were stationary, that is to say, orthodox teaching supports the point A′ instead of the point B, and makes the light take the 45° track from A′, in spite of the relative motion; whereas the point B has the advantage of lying on the same occurrence line as A, while the light-track is sloped at an angle corresponding to the combined velocities c and u.

We being the moving observers observe the event at S′, and we have the option whether we infer that it occurred at A′ or at B, *i. e.* (allowing for the FitzGerald contraction) at A; this is the place at which the testimony of a stationary observer would concur.

We will draw the diagram simply as fig. 7. One ought first of all to think of the definite hyperbola $x^2-y^2=\text{const}=\text{OA}^2$. This gives O and x_0. Then for a given u, one can get x' by drawing AM at the angle $\alpha=\sin^{-1}\frac{u}{c}$, and then,

since $$x'^2-y'^2=\text{OA}^2,$$

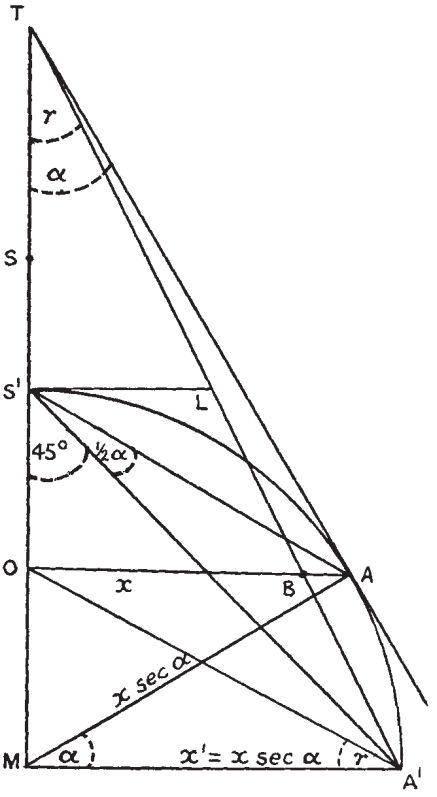
it follows that $x'^2 = OA^2 + OM^2$;

$= MA^2$;

$\therefore \quad MA' = MA$;

so practically we use a circle centre M, radius MA, to find A′ the desired point on the hyperbola. Then a tangent at A′ will give B, and the time which elapses

Fig. 7.



before the event A is seen is $\frac{OB}{c+u}$. There are several interesting pieces of geometry connected with the figure :—

(1) The line AT⊥to MA is at angle α with MOT, so TA′ is at angle γ, since

$$\tan A'TM = \frac{MA'}{MT} = \frac{MA}{MT} = \sin ATM = \sin \alpha = u/c.$$

(2) OA′M=γ, since its tangent=$\frac{OM}{MA'}=\frac{OM}{MA}=\sin\alpha$.

(3) A′T, A′O are equally inclined to the 45° line A′S′. Therefore, S′L (in fig. 7)=S′O, the triangles S′LA′, S′OA′ being congruent.

Or, putting it another way, there is a displacement of Jupiter due to relative velocity u so that its sloping line touches the hyperbola $x^2-(ct)^2$=constant instead of just sloping from the point $(x, 0)$ and therefore crosses the "now" line OA at a forward point B such that OB=OA cos α (FitzGerald contraction).

Now the eclipse effect reaches the eye in the time $\frac{x'+ct'}{c}$ from its actual stationary place at A$(x, 0)$. This time is equal to $\frac{x\cos\alpha}{c+u}$: *therefore* we have the right to choose between two alternatives:

(1) The eclipse seems to occur earlier and further away (at A′) and the velocity of approaching light appears as just c and no more: *or*

(2) The eclipse occurs at time $t=0$, but by reason of relative motion the apparent distance is contracted into $x\cos\alpha$, and the light approaches with velocity c, but meets the observer with *relative* velocity $c+u$; in the time $\frac{x\cos\alpha}{c+u}$.

We have to choose between these two alternatives, and we are inclined to think the extended FitzGerald contraction the less violating to common sense.

Caution.

But the contraction is generally supposed to apply to matter, and to be due to its relative motion. What the significance of applying it to space not in motion may be, demands further consideration. There is no doubt, however, that the travel of an observer with velocity u towards the light ought to make him perceive that it is moving relatively to him with the velocity $c+u$, and the contraction seems to follow as a kind of effect analogous to aberration.

We must remember that for *successive events* on Jupiter, the observer's track is up the line OT, and there are

different hyperbolæ, each event A being at the vertex, and A′ a point on it such that Jupiter's time-path touches the hyperbola at A′.

Every A will be on the line TA. Every A′ and every B will be on the line TA′. Also AOA′ is always γ, so as A is known, A′ is drawable by means of γ, *i. e.*, the OA′ lines are all parallel.

The advantage of B over A′ is that it does not alter the time of an event, but only contracts OA. The advantage of A′ over B is that it gives us quickly the time interval $\frac{MS'}{c}$, by means of MS′=MA′, using the 45° line from A′ to S′, S′ being the sight moment.

Note.—B is no help to the geometry, it just waits patiently for someone to pay attention to it.

LVII. *Notices respecting New Books.*

Origine des Rayons Gamma. Structure fine du spectre magnetique des Rayons alpha. By SALOMON ROSENBLUM. No. 4 of Exposés de Physique Théorique under the direction of M. Louis de Broglie. [35 pages quarto.] (Paris: Hermann et Cie. Price 12 fr.)

THIS little monograph contains a concise account of the experimental work on the fine structure of alpha rays, in particular the work with the Paris electromagnets; and these experimental results are linked with the energy level schemes suggested by Gamow to explain the emission of gamma radiation. It is admirably arranged and printed, and since this work is one of the remarkable successes of commonplace ideas boldly applied to nuclear problems it should have a wide appeal to all physicists, not only to the specialist.

L'Existence du Neutron. By IRÈNE CURIE et F. JOLIOT. No. 2 of Exposés de Physique Théorique under the direction of M. Louis de Broglie. [22 pages quarto.] (Paris: Hermann et Cie. Price 6 fr.)

THE two authors have here collected and set out an account of the series of experiments with ionization chamber and expansion chamber, which led to the discovery that the artificially excited nuclear radiation can project other nuclei.